SDI LAB #6. NEWTON'S SECOND LAW REVISITED†

NAME ______________________________ _______________________ ______ ______ ______
Last (Print Clearly) First (Print Clearly) ID Number

LAB SECTION _______________________LAB TABLE POSITION___________________________

> *One afternoon several years ago the writer was asked to proctor an examination in elementary physics to be administered to a large room full of army trainees. As he strolled the room waiting for the examination to begin he overheard many snatches of excited, apprehensive conversation – of which one significant piece has haunted him ever since: "Sure, I know F = ma, but what's F? what's m? what's a?"*
>
> Robert Weinstock, *Am. J. Phys.* **29**, 698 (1961)

I. INTRODUCTION

In this lab you measure the output of a "force probe" versus the acceleration for a cart with light, medium, and heavy loads as shown in Fig. 1 on the next page. For each load the cart is subjected to various constant accelerations. You analyze the data and probe the essential meaning of Newton's Second Law $\vec{\mathbf{F}}_{\text{net on body}} = m_{\text{body}}\vec{\mathbf{a}}_{\text{body}}$ by carefully addressing the crucial questions:

"What's F? What's m? What's a?"

A. OBJECTIVES

1. To use a sonar motion detector to measure acceleration; a Hall-effect-leaf-spring probe to measure force; and a computer to obtain, analyze, and plot these data.
2. To interpret the data in terms of *operational* definitions of $\vec{\mathbf{F}}$, m, and $\vec{\mathbf{a}}$.
3. To consider the relationship between inertial and gravitational mass.

B. HOW TO PREPARE FOR THIS LAB

1. Review SDI Labs #0.1 and #2 with particular attention to your definitions of kinematic terms. Review SDI Lab #1 with particular attention to your formulation of a non-rigorous operational definition of "force."
2. Study the P201 text, *Physics*, 4th ed. by D. C. Giancoli (or similar material in whatever text you are using): Chap. 4 on "Motion and Force: Dynamics"; p. 107 - 110 on gravitation; p. 948 - 949 on gravitational and inertial mass.

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†SDI Lab #6, RW & RRH, 1/12/98. Based on a "Newtonian sequence" treatment by A. B. Arons in *A Guide to Introductory Physics Teaching* (Wiley, 1990), p. 52-55. The force probe, sonic motion detector, computer tools, and software were developed at the *Center for Science and Mathematics Teaching* at Tufts University.

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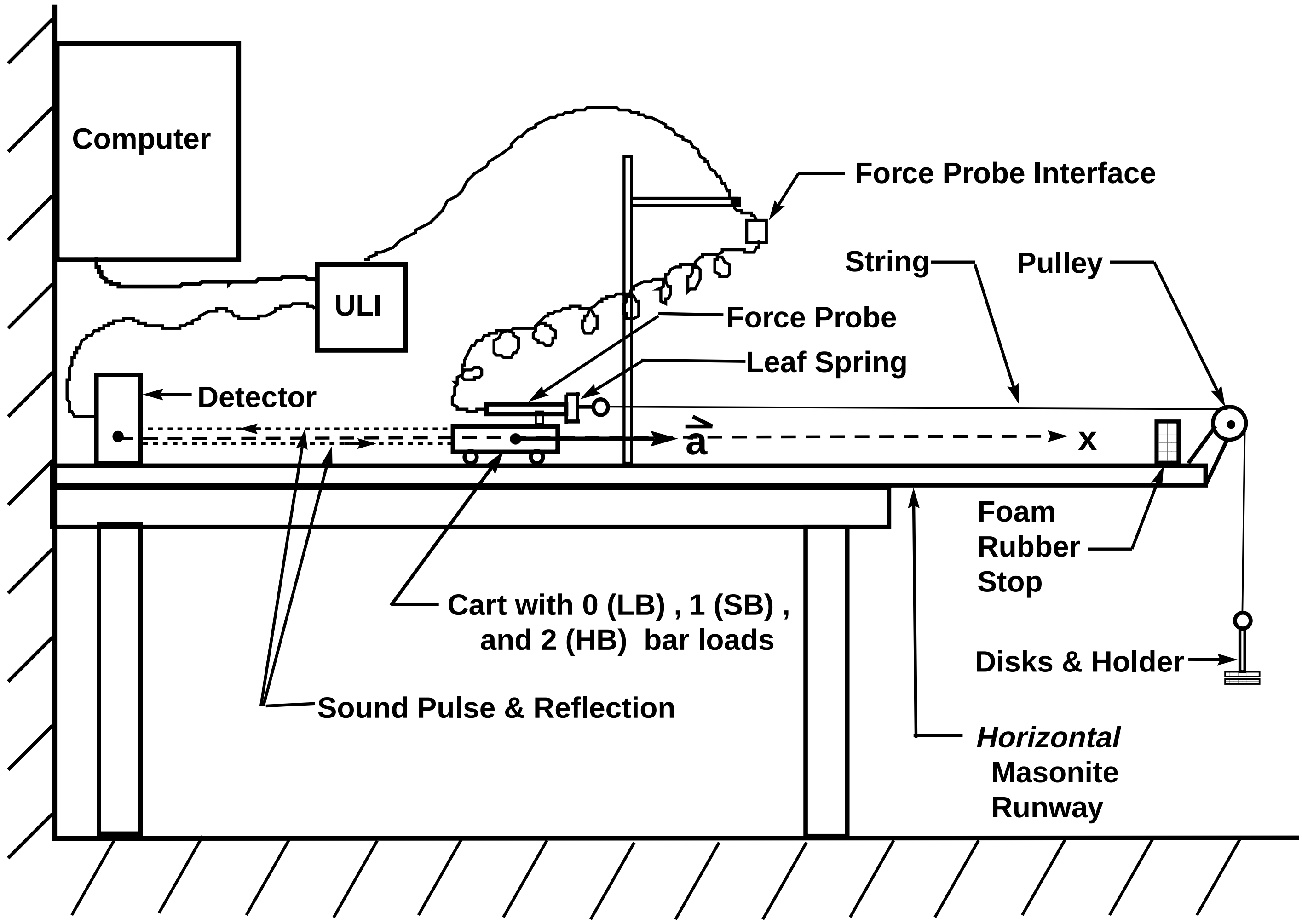

Fig. 1. The apparatus for determining acceleration and Force-Probe Output (FPO) for different constant accelerations, a, of a cart on a *horizontal* runway. The different accelerations are produced by varying the number of disks on the suspended holder. The cart is loaded first with one (Standard Body ≡ SB), then two (Heavy Body ≡ HB), and then zero (Light Body ≡ LB) loading bars. The computer is a MacIIsi running MacMotion 4.0. The Universal Lab Interface (ULI) is from Vernier Software.

The time interval for the reflection and return of the motion detector's sound pulse yields x(t), i.e., the cart position x versus time. From x(t) the computer software calculates v(t) as $\Delta x/\Delta t$ and then a(t) as $\Delta v/\Delta t$. The x(t) data are collected at a rate of 20 points per second.

The tension T in the string deflects a leaf spring by some amount δ. This, then, causes an outward movement of a small magnet attached to the spring. Movement of the magnet results in the reduction of the magnet field applied to a semiconductor slab (visible under the spring). This, in turn, causes a decrease in a voltage ("Hall Effect") across the slab, interpreted by the (interface + computer) as proportional to a force through Hook's law $F = -k\delta$. The Hall Effect voltage as processed by the interface constitutes the Force-Probe Output.

Acceleration (a) and Force-Probe Output (FPO) are displayed in nearly real time on the computer screen. The data are graphed and analyzed by the computer software.

II. HOW TO DO THIS LAB

This lab involves *five* different activities using the apparatus of Fig. 1. *You will do activities B and C (one right after the other)* ***three*** *times for three different carts (called SB, HB, and LB)* ***before*** *going on to step D.*

A. Launch the computer programs.

B. Take and record in Table I, p. 6, the Force Probe Output (FPO) versus acceleration, a, data for one particular loaded cart for three different values of the acceleration produced by three different hanging-disk systems.

C. *Display* a graph on the computer-screen of (FPO) versus acceleration, a, for the three accelerations of that cart and answer questions regarding that graph.

D. *Print out* a graph for each lab partner of the (FPO) versus acceleration, a, data collected in "C" for the three differently loaded carts and interpret these data.

E. Display and then print for each lab partner a graph of *gravitational* mass m_g versus the slope of the FPO vs a curves for three differently loaded carts and interpret these data.

III. LAUNCH THE COMPUTER PROGRAM (this is a set of procedures that *you do just once* at the start of the lab)

A. If the computer equipment is not on, then turn it on [computer, Universal Lab Interface (ULI) box, and printer].

B. Open the folder called "N's 2nd exp." If it is not open already, open it by double clicking on it.

C. Select all the files in this folder by typing "command A" (That is, *hold down* the Apple "command key" marked ⌘ and then type "A". Henceforth we shall use the symbol ⌘ to stand for "command.")

D. Open these files by typing ⌘O. The computer will spend some time launching programs, eventually ending with the *MacMotion* program.

E. Experiment a bit with the cart set-up: Drape the string over the pulley and then determine the range of motion of the cart. Note that *even with the foam rubber stop in place it is a good idea to slow the cart near the end of its motion so that it will not jump the barrier and crash to the floor!*

F. Experiment with the force probe. First you must "zero" the probe. Do this by holding the force probe and the string which is attached to it ***so that the string is slack***. Then type ⌘B on the keyboard. Zeroings take a few seconds; the cursor arrow will turn into a little picture of a watch while you are waiting. When it turns back into an arrow you are ready to proceed.

Now hit the ***Return Key*** on the keyboard. *Gently* pull on the force probe hook while the motion detector is clicking. (The probe is set to measure *very* small forces.) Watch the FPO(t) curve go up and down as you do this. The apparatus is set up to collect data for only 2 seconds so you may want to repeat this experiment a few times.

G. Select **Analyze Data A** from the **Analyze** menu. (To do this, point to the word **Analyze** at the top of the screen and click the mouse button *and hold it down*. A menu will pop down. Move the mouse until the words **Analyze Data A** are darkened, then let go of the mouse button.)

IV. TAKE PROBE OUTPUT (FPO) vs ACCELERATION DATA FOR THREE CARTS

- The first time you go through this procedure you should have *one* of the black bars loaded onto the cart. The cart loaded with *one* black bar will be called the "Standard Body" ≡ "SB".
- When you have completed the steps in this Sec. IV for the cart SB , go to **Sec. V** "**Display A GRAPH......"** and then **Sec. VI** "**QUESTIONS TO ANSWER**...."
- After you have displayed a graph and answered the questions, *you will come back to this Sec.* IV, only this time you will put *both* of the bars on the cart. Call this cart the "H-BODY" ≡ "HB" ("H" stands for "Heavy"). After you take data for this cart, you will go again to Sections V and VI.
- The third time you do this Sec. IV, you will remove both the bars so that there are *no* bars on the cart. Call this cart the "L-Body" ≡ "LB" ("L" stands for "Light"). When you complete this section go again to Sec. V and VI.

For each body, you will take three sets of data, each one for a different acceleration of the cart. The acceleration will be caused by the hanging disks. Three disk systems will be used (a) the disk holder alone , (2) the disk holder plus one additional disk, and (3) the disk holder plus two disks. *Start with the unloaded disk holder.*

A. Before you collect data, you need to zero the force probe. **ZERO THE FORCE PROBE (with the string limp) by typing B PRIOR TO EACH AND EVERY DATA COLLECTION RUN!!**

B. Locate the cart at the "START" position with the black plastic disk at the back of the cart just above the penciled outline on the runway. This will place the hanging disk is at its highest position. *Be sure that the runway is clear so that the sound pulses will not reflect from objects other than the cart or from you or your lab partners.* It is a good idea to sweep any debris from the runway before starting this experiment. The masonite runway has been carefully leveled by your instructors so please don't shift it around.

Push the ***Return Key***. *As soon as you hear clicks from the motion detector*, release the cart. Be sure that the foam rubber stop at the end of the runway is in place. As previously indicated, *even with the foam rubber stop in place it is a good idea to slow the cart near the end of its motion so that it will not jump the barrier and crash to the floor!*

C. As the cart moves, graphs will appear on the screen. The graphs will be of the acceleration of the cart vs time, and of the Force Probe Output [simply labeled "Force (?) "] vs time. The graphs should look something like those on the next page. ***If they do not then please consult with an instructor.***

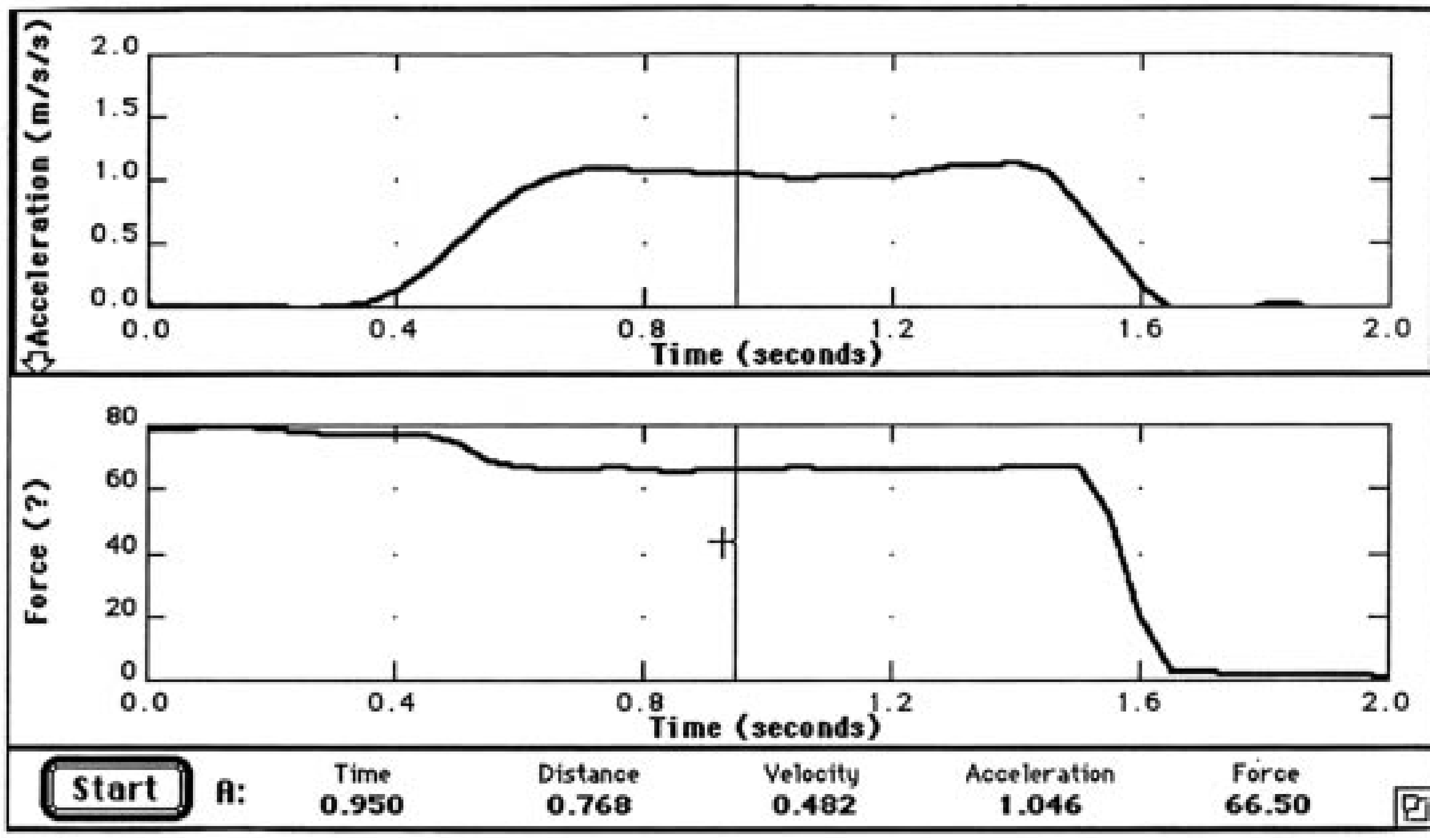

Fig. 2. Typical acceleration and Force Probe Output (FPO) data as seen on the computer screen of Fig. 1. These data were obtained for the cart loaded with one bar (Standard Body) and with two disks on the disk holder. (Note the initial reduction in the FPO as the cart goes from a stationary to an accelerating condition.) The rounding of the a(t) curve at the start and end of the run is due to the computer's data-averaging of the v(t) and a(t) data. We do not presently understand the cause of the slight dip in the plateau region of the a(t) curve and other occasional eccentricities in the a(t) and FPO(t) curves.

Since the Analyze tool is active, you can pinpoint the acceleration and force at a particular time. As you move the cursor arrow in the graph window, a vertical line moves across the graph. *Use your judgement* to set the line at a time, t, so as to read off a roughly average value of acceleration, a, for the plateau region of the a(t) curve, and then record this average a and its corresponding FPO (unfortunately called "Force" by MacMotion) in Table I below. For the case shown, the acceleration is 1.046 m/s^2 and the FPO is 66.50. The FPO is in some arbitrary force-probe units (fpu), whose calibration is considered latter in the experiment.

Note the *qualitative* variation of the distance (i.e., position), velocity, acceleration, and FPO numbers at the top of your graph (similar to Fig. 2) as you scan the data by moving the vertical line along the nearly flat plateau region of the FPO vs time curve. Do the observed variations support the Aristotelian view that *velocity* is directly proportional to force? {Y, N, U, NOT}*****

*****Please recall the and abide by the ground rules for SDI labs set forth in SDI Lab #0.1: A curly bracket {.......} indicates that you should **ENCIRCLE** O a response within the bracket and then, we **INSIST**, briefly **EXPLAIN** or **JUSTIFY** your answers in the space provided on these sheets. The letters { Y, N, U, NOT} stand for {Yes, No, Uncertain, None Of These}.

D. Record the average acceleration and average FPO in the appropriate row of the table below.

Table I. Average acceleration and average Force-Probe Output data.

Cart	**Number of Disks Added To Holder**	**Average Acceleration** (m/s^2)	**Average FPO** arbitrary units called (fpu)	**Comments**
SB	**0**			
SB	**1**			
SB	**2**			
HB	**0**			
HB	**1**			
HB	**2**			
LB	**0**			
LB	**1**			
LB	**2**			

E. Repeat this procedure, adding first one disk, and then two disks to the disk holder. Enter the data in Table I above. You may wish to make comments on the data in the "Comments" column of the data.

NOTE: *Please follow good laboratory practice.* If you retake some of the data, then place a *line* ——— through the earlier data but ***do NOT erase it.*** On later analysis, the earlier data may turn out to be correct or may disclose valuable information.

V. DISPLAY A GRAPH OF THE FORCE PROBE OUTPUT (FPO) VERSUS ACCELERATION FOR A CART

For this section, you will need to use another computer program called *Graphical Analysis*. It will produce a graph of the force and acceleration data which you enter into it. It should speed up your analysis of the data, thus allowing you more time to consider the physics.

A. To get to the *Graphical Analysis* program, point to the small icon in the extreme upper-right corner of your computer screen. Click on the icon and hold the mouse button down. Move the cursor down until the words *Graphical Analysis* are darkened. Then let go of the mouse button.

B. The *Graphical Analysis* program will appear on the screen. There should be *four* windows: three small data windows on the left and one large graph window on the right. To enter data, click on one of the graph windows. When you go through this procedure the first time, you will want the *top* data window, labeled S-Body. (Use the next window down when you get there the next time.)

Double click in the space below **Acc**, **m/s^2** and just to the right of the row number **1**. You can now type in the first acceleration from Table I. Pushing the ***Enter Key*** on the numerical key pad will move the active box to the right, and you can type in the corresponding FPO. (If you ever need to go back to change something, you can always point with the mouse and double click. The arrow keys also work well.)

Enter the three rows of data from your table.

C. As you enter data, the points will be plotted automatically in the graph window. A best fit line will also appear. In the space below the graph are some statistics about the line.

D. If you wish to print your graph, select the graph window by clicking once anywhere on the graph window. Then type P. (Normally, it’s best to wait until the data for all three loadings of the cart have been obtained before printing out any graphs as indicated in Sec.VI-C4 below.)

VI. QUESTIONS TO ANSWER AFTER OBTAINING FPO vs ACCELERATION DATA FOR A CART

A. After plotting FPO vs acceleration data for the *first* cart (SB):

1. From the appearance of the graph, would you say that the data points for SB lie approximately along a straight line? {Y, N, U, NOT}

One measure of how close points are to a straight line is the "Correlation Of Regression," listed under "C.O.R." below the graph. The closer this number is to 1, the more linear the data. What is the C.O.R. for your SB line? _________________.

2. Does the SB line go approximately through the origin? [The y-intercept is ________________fpu (force probe units).] {Y, N, U, NOT} [HINT: How large is the intercept in relation to the maximum FPO reading? (express this as a percentage___________%).]

B. After plotting FPO vs acceleration data for the *second* cart (HB):

1. From the appearance of the graph, would you say that the data points for HB lie approximately along a straight line? {Y, N, U, NOT} (The C.R.O. is ___________)

2. Does the HB line go approximately through the origin? {Y, N, U, NOT}
The y-intercept is _______________________fpu.

3. Check one: is the slope for HB larger____, smaller_____, or the same_____ as for the Standard Body?

C. After plotting FPO vs acceleration data for the *third* cart (LB):

1. From the appearance of the graph, would you say that the data points for LB lie approximately along a straight line? {Y, N, U, NOT} (The C.R.O. is ___________)

2. Does the LB line go approximately through the origin? {Y, N, U, NOT}
The y-intercept is _______________________fpu.

3. Check one: is the slope for LB larger____, smaller_____, or the same_____ as for the Standard Body?

4. *Print out* graphs for the SB, HB, and LB bodies. First select the graph window by clicking once anywhere on the graph window. Then type P and either hit the ***Return Ke***y or click with the mouse on "Print" in the window. Do this n times so as to obtain n graphs for the n partners in your group.

In order to make the physical interpretation of the graph (Sec. VII) easier, please *label* the three curves, as, e.g., "Heavy Body," "Standard Body," "Light Body."

5. Cut the graphs out of the 8.5 x 11 inch sheets and ***tape one of these to the top of the next page.***

VII. INTERPRETATION OF THE DATA

A. Rough Qualitative Interpretation

1. Before attempting a detailed interpretation of the FPO vs acceleration graph above, can you give a *rough qualitative* interpretation? {Y, N, U, NOT}

   In answering this question please analyze the following three aspects of the graph:

   (1) FPO vs acceleration a for *one* cart:

2) FPO *at some constant acceleration* a for the three differently loaded carts:

(3) Acceleration a *at some constant* FPO for the three differently loaded carts:

2. Does the above graph support the Aristotelian view that *velocity* is directly proportional to force? {Y, N, U, NOT}

B. *What's* a*?*

1. Can you give an ***operational*** definition of acceleration $\vec{\mathbf{a}}$? {Y, N, U, NOT} [HINT #1: You may wish to review your previous work on operational definitions in SDI Lab # 2, *Newton's Second Law*. HINT #2: What is the mathematical definition of acceleration in terms of a limit? HINT#3: Operational definitions always specify *measurement* methods.]

2. As discussed in the caption to Fig. 1, the computer can convert the position vs time data to display velocity as a function of time in accord with $v = \Delta x/\Delta t$. Then the computer can convert the velocity vs time data to display acceleration as a function of time in accord with $a = \Delta v/\Delta t$. Do you think such computer plots are consistent with operational definition of *acceleration* given by you in "1" above? {Y, N, U, NOT}

C. *What's* F?

1. We have tacitly assumed that frictional forces can be ignored in this experiment so that the force probe output is proportional to the *net* force acting on the carts. That friction can be ignored might seem reasonable considering the very low–friction carts that are used. (Note that friction in the pulley should have little effect on the FPO(a) curves since the force probe measures only the tension in the horizontal string.) Do the present data offer any support for this assumption? {Y, N, U, NOT} [HINT: Suppose a kinetic frictional force $\mu_k(m_c g)$ acted on the cart of mass m_c . What effect would this have on the FPO(a) curves?]

2. You have designated the cart loaded with one black bar as the "Standard Body." Your graph shows Force Probe Output (FPO) readings vs the acceleration a for the standard body. Can you now operationally define a "Force" scale in terms of the operationally defined acceleration of the Standard Body? {Y, N, U, NOT} [HINT: Suppose that you had used as the standard body the platinum-iridium cylinder kept at the International Bureau of Weights and Measures whose mass is by definition precisely one kilogram (see e.g., Giancoli, p. 68, or similar material in whatever text you are using). What then would have been your force scale units corresponding to accelerations of 1 and 2 m/s^2? (please fill in: ________________________________).]

D. *What's m?*

If your results are similar to those obtained by other students with this apparatus, then in Sec. VI you concluded that the data for the *three* carts (SB, HB, and LB) yield *three* straight lines with *three* different slopes for the Force Probe Output (FPO) vs. acceleration. Would you be justified in saying, then, that the F vs a slope might (with further testing) turn out to be *a unique property of every body ?*
{Y, N, U, NOT}

If your answer above is "Yes" and you gave a good justification then go on to question "E" below. If not then please discuss the matter with an instructor.

E. *What's "F = ma"?*

Suppose you *assume* that the F vs a slope is a unique characteristic of every body in the universe, and designate the slope as the "*inertial* mass $\equiv$ m." You could then summarize the results of the present investigation by the equation "F = ma". Suppose you had done these experiments in 1665, well before Newton's work, and suppose that your "F = ma" hypothesis had been confirmed by others throughout the world.

1. Do you think your "F = ma" relationship would have been justifiably hailed as a "new law of nature"? {Y, N, U, NOT}

2. Do you think your "F = ma" relationship could have been justifiably dismissed on the grounds that "F = ma" is "circular"? (Here "circular" means that "F = ma" is true only because F and m are *defined* in such a way as to automatically make F = ma without yielding any new information on the behavior of nature.) {Y, N, U, NOT}

F. *Is "F = ma" the whole story?*

In "E" above, suppose you are invited to lecture before the *Royal Society of England* on your work showing that "F = ma." You decline, stating that *the road to knowledge is paved with questions* and that such questions need to settled by experiment before "F = ma" can be usefully employed. Do you have any idea as to what questions might have been on your mind (assume that you are even smarter than Newton)? {Y, N, U, NOT} [HINT: Think of the mechanics lore beyond the scalar "F = ma," that you must apply in order to solve a typical mechanics problem.]

III. GRAVITATIONAL AND INERTIAL MASS

A. The Inertial Masses of the Carts

We can place the Standard Body cart on a spring balance and measure thereby the force $\vec{\mathbf{F}}_{\text{on cart by Earth}}$. We could then obtain the *gravitational* mass $m_g(SB)$ of the standard body from Newton's Universal Law of Gravitation (NULG) as

$$m_g(SB) = F_{\text{on SB by Earth}}\,[R_E^2 / (G\,M_E)] = F_{\text{on SB by Earth}}\,(\beta) \qquad \text{..........................(1)}$$

where G is the gravitational constant, M_E is the mass of the Earth, R_E is the radius of the earth, and $\beta \equiv [R_E^2 / (G\,M_E)]$ is a constant.

Do you know the numerical value of the constant β in Eq. (1)? {Y, N, U, NOT}
[HINT: You should not need to calculate $R_E^2 / (G\,M_E)$.]

We could also use NULG to obtain $m_g(SB)$ by measuring the gravitational force of attraction $F_{\text{on SB by } M_s}$ between SB and any standard mass M_S. This would, of course, require a very sensitive balance [see Giancoli, p. 120 for a picture of a sensitive torsion balance used to make such measurements (or a similar picture in whatever text you are using).]

From data based on NULG equations such as (1), we know the *gravitational mass* m_g of the carts SB, HB, and LB. Is there any obvious reason why the *gravitational* mass m_g as defined by an equation such as (1) should be proportional to the the *inertial* mass m_i defined as the slope of an F vs a curve? {Y, N, U, NOT} [HINT: For a discussion, see Giancoli, p. 948 - 949, or similar comparison of gravitational and inertial mass in whatever text you are using.]

B. Comparison of Inertial and Gravitational Masses of the Carts

1. Launch a new arrangement of the graphing program.
From within the *Graphical Analysis* program, first close the current graph by typing ⌘W. When a box appears asking you if you want to save changes, ***CLICK ON THE "NO" BUTTON !!*** Now get the *Open Dialogue Box* by typing ⌘O. Double click on **F/a vs. m folder** and then on **F/a vs. m graph.**

2. Display and print the gravitational mass m_g vs the inertial mass m_i (the slope of the FPO vs a curve) for each cart.

a. The values of m_i (FPO vs a slope) are listed in the statistics below the graph you printed in Sec. VI-C4 (the graph obtained after you obtained data for the LB cart). Please list these values here, all in [FPO units/(m/s^2)]: m_i(SB) =_______, m_i(HB) = _______, m_i(LB) =_______.

b. Calculate and list below the m_g values for SB, HB, and LB carts from the facts that (1) the gravitational mass of the cart alone is 0.632 kg, and (2) each black bar has a gravitational mass of 0.498 kg: m_g(SB) = _________kg, m_g(HB) = _________kg, m_g(LB) = _________kg.

c. Enter the data in "a" and "b" into the graph program so as to plot m_g vs m_i. Print the graph n times for each of the n partners in your group.

d. Cut the graphs out of the 8.5 x 11 inch sheets and **tape one of these at the top of the next page**.

C. Interpretation of the Inertial and Gravitational Mass Data

1. From the appearance of the above graph, would you say that the data points for m_g vs m_i lie along an approximately straight line? {Y, N, U, NOT} (The C.R.O. is ___________).

2. Does the m_g vs m_i line go approximately through the origin? {Y, N, U, NOT}
The y-intercept is _______________________kg.

3. What can you conclude from your results in "1, 2" regarding the relationship of m_g and m_i?*

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*For a good presentation of experiments testing the equivalence of inertial and gravitational mass and the relevance of this equivalence to the general theory of relativity see, R. H. Dicke, "The Eötvös Experiment," *Scientific American,* December 1961.