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THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE HONORS DEGREE OF

BACHELOR OF SCIENCE IN PHYSICS

Faraday Cup Design and Geant4 Simulation for the ALPHA Project at IUCF

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December 10, 2009

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1 Abstract

Faraday cups present a means of obtaining a highly accurate and direct, though destructive, measurement of charged particle beam current and are commonly used in the accelerator physics community. This paper includes an in depth description of the Geant4 simulations that were used to design the Faraday cup that will be used as a current monitor and emergency beam dump in the ALPHA project’s electron accelerator. Additionally, a summary of Faraday cup construction and operation considerations is provided.

2 Introduction

In any experiment involving a charged particle beam, the intensity of the beam is of fundamental importance. Beam intensity dictates the type of activities that can be performed, as well as the basic limitations on the types and accuracy of the measurements that can be acquired. For this reason, the charged particle beam current, being directly related to the beam intensity, is an important parameter to monitor and control. To this end, a Faraday cup is commonly used to obtain an absolute beam current measurement in a variety of experimental settings, including particle accelerators such as electron storage rings.

The primary objective of this paper is to describe the simulation and design of a Faraday cup system to be used as a current monitor and emergency beam dump in the extraction line of the Advanced eLectron PHoton fAcility (ALPHA) project’s electron storage ring that is currently under construction at the Indiana University Cyclotron Facility (IUCF). The ALPHA project was commissioned to assist the Naval Surface Warfare Center, Crane Division (Crane NSWC) in modernizing their radiation survivability testing capabilities while concurrently providing IUCF with advanced research opportunities in accelerator physics, including the development of a compact x-ray light source using inverse Compton scattering [1] and a Free Electron Laser (FEL), and condensed matter physics.

The simulation of the Faraday cup was carried out using the Geant4 particle simulation toolkit [2]. Geant4 was developed by an international scientific and computing collaboration organized by
the European Organization for Nuclear Research (CERN) to facilitate the creation of physically accurate and computationally efficient Monte Carlo simulations of modern high energy physics experiments such as the detectors used at the Large Hadron Collider (LHC). Subsequently, Geant4 has been employed as a powerful research tool in a variety of fields including nuclear physics, medical physics, and space physics [3]. Designed as a successor to GEANT3 which utilized aging FORTRAN technology, Geant4 represents a complete rebuild in C++ using thoroughly modern object oriented software engineering principles [4].

A secondary objective of this paper has been to collect and organize some of the design ideas and operational experience related to the implementation of Faraday cups in a variety of experimental situations. The text of this paper and its references represent a fairly comprehensive introduction to Faraday cups with details that are relevant to accelerator systems with a broad range of designs and power ratings.

3 ALPHA Project Description

As a joint collaboration between IUCF and Crane NSWC, the primary goal of the ALPHA project is to create a compact and cost-effective electron accelerator system that can serve as a flexible radiation dosage source for survivability testing while simultaneously providing new research opportunities at IUCF for the accelerator physics group led by S.Y. Lee and for condensed matter physics.

3.1 New Features of ALPHA

In terms of radiation survivability testing, ALPHA represents a dramatic step forward toward new and exciting technologies. In addition to supplying a high instantaneous radiation dose rate of around $1 \times 10^{12}$ rad/s, ALPHA introduces the following new features:

1. Microstructure debunching: Due to the RF energy amplification used in linear accelerators, the operation of a linac typically produces a macroscopic beam with a microstructure that has a frequency around 1 GHz. When the Crane NSWC linac facility began performing
Figure 1: The ALPHA Electron Accelerator
radiation survivability tests in the late 1980’s, this was not an issue since electronics of the
day used clock rates that were orders of magnitude slower. However, in accordance with
Moore’s Law, microprocessors with clock speeds of 4 GHz and above are now common place
and so interference effects with the RF signal can become a problem.

To eliminate this concern, the ALPHA electron storage ring uses momentum compaction
factor tuning to shift the phase space of the beam so that the micropulses overlap, as illustrated
in Figure 2. This effectively debunches the RF microstructure of the beam, resulting in a
continuous beam with a small ripple, specified to be below ±10%.

![Figure 2: An illustration of ALPHA’s micropulse debunching](image)

2. Nonlinear Beam Spreading: In radiation survivability facilities that use an electron beam as
a dosage source, the beam profile is typically Gaussian to a first order approximation. In this
case, a metal collimator is often used to cut off the tails of the distribution, resulting in a
more uniform radiation dose profile. However, the use of a collimator introduces a significant
amount of scattering around the sample. This may result in a loss of precise dose rate control
over the area of the sample being tested, as well as a higher than desired background radiation
level in portions of the sample that are not undergoing testing.

Through the use of octuple magnets, the extracted beam in ALPHA will be shaped so that it
obtains a uniformly distributed rectangular beam profile. By shaping the beam profile using
accelerator optics techniques, the need for a collimator is eliminated. The simulated radiation
dose profile through a potential Device Under Test (DUT), represented by a 2 cm × 2 cm ×
0.15875 cm silicon slab, in the ALPHA test area is shown in Figure 3.

Figure 3: Simulated sample dose profile in the ALPHA test area

3. Clean Radiation Test Environment: In addition to the use of a collimator, typical radiation survivability facilities use test fixtures that place the beam stop directly behind the sample being irradiated. This further increases the background radiation dosage. Assuming this background level is accurately monitored through dose rate measurements, the error it causes can be taken into account. However, scattering off the collimator and beam stop can lead to the irradiation of portions of the sample that the user does not want to expose to radiation, and this effect can be difficult to shield.

In ALPHA, the absence of a collimator coupled with the placement of the beam stop several feet away from the sample position allows for a very clean radiation test environment with a background radiation level of about 0.1% of the total delivered radiation dose. Figure 4 shows the results of a Geant4 simulation of the ALPHA test area with 200 incident electrons at 50 MeV. Here, the simulated silicon sample described above is positioned atop a stainless steel sample table, located near the top of the figure. This is placed at a reference height of
36" above the beam stop enclosure consisting of a 24" × 60" concrete-lined floor hole that houses a lead Faraday cup, which is separate from, though influenced by, the Faraday cup described in this paper. Note that essentially none of the incident electrons, or any other massive particles, are backscattered through the sample while only a very small portion of the gamma rays produced in the collision pass near to the sample. (It should be mentioned that these 200 incident electron simulations are intended solely as a visual illustration of the radiation test environment. Such illustrations can be enlightening, but, as discussed below, a simulation this small is ultimately useless for quantitative measurements.)

![Image of Massive particle trajectories and Photon trajectories.](image)

(a) Massive particle trajectories. Electrons are red and neutrons are green. (b) Photon trajectories.

Figure 4: Simulated ALPHA radiation test area

### 3.2 ALPHA Modes of Operation

The ALPHA electron accelerator is designed to be used under two different modes of operation:

1. Long Pulse Mode (Single Pass Operation)
2. Short Pulse Mode (Transient Operation)
In the Long Pulse Mode, the macropulse from the linear acceleration moves through the electron storage ring a single time, becomes debunched, and then exits to the extraction line. In this case, the timing parameters remain unchanged and the pulse length and the repetition rate are controlled directly by the linac. In this mode, the maximum instantaneous dose rate is $0.4 \times 10^{10} \text{ rad/s}$ over 6.45 cm$^2$ for a pulse length of 4 $\mu\text{s}$.

In the Short Pulse Mode, the macropulses sent from the linear accelerator are accumulated in the electron storage ring to a maximum macroscopic pulse charge of 1000 nC over a period of around 10 minutes. Using an RF barrier bucket, the pulse length can be condensed to a minimum of 25 ns, which yields a maximum instantaneous dose rate of $1 \times 10^{12} \text{ rad/s}$ over 16 cm$^2$.

### 3.3 ALPHA Electron Accelerator Description

As shown in Figure 1, the ALPHA project’s electron accelerator consists of three main subsections:

1. **Injection Line**

2. **Electron Storage Ring**

3. **Extraction Line**

The injection line consists of a linear accelerator followed by a sequence of dipole and quadruple magnets designed to deliver the beam to a Lambertson septum where the electrons are then injected into the appropriate phase space in the electron storage ring. As the ALPHA project progresses, the linac used in the injection line will become more powerful during different stages of operation. The initial commissioning phase will use a Varian Clinac (shown in Figure 1), an electron linear accelerator typically used in the medical sector for radiation therapy. This linac has the capability to produce 20 MeV electrons with a beam current of 20 mA. Upon the successful completion of the initial commissioning phase, a more powerful linear accelerator will be built adjacent to the ALPHA facility, possibly by placing several Varian Clinacs in series, which will produce 50 MeV electrons with a beam current of 68 mA. During both phases, the linear accelerators will output a maximum pulse length of 4 $\mu\text{s}$ at a repetition rate of 10 Hz. These timing parameters are required to allow for sufficient beam cooling within the electron storage ring.
In order to maintain the desired extracted beam parameters, the electron storage ring requires a particle energy spread of no more than 1-2%. In order to achieve this, copper beam slits will be inserted into the beam pipe directly following one or both of the bending magnets in the injection line. These beam slits place a definitive physical restriction on the horizontal transverse emittance of the electron beam. Since, as in a spectrometer, the magnetic field of the dipole magnet horizontally spreads out the electron beam as function of particle energy, the beam slit width can be used to determine the energy spread of the electron beam.

The electron storage ring possesses a beam line circumference of approximately 20 m and is maintained at an ultrahigh vacuum level of $10^{-11}$ torr to facilitate the long storage lifetime required by the Short Pulse Mode. Some of the electron storage ring components include:

1. The four main dipole magnets which were recomissioned from IUCF’s former Cooler Injector Synchrotron (CIS).

2. The Lambertson septum which is used for both injection and extraction.

3. Magnetic and electrostatic kickers which are used to bump the beam into the Lambertson septum’s acceptance orbit for extraction.

4. A gradient damping wriggler that will allow for the momentum compaction factor tuning required for microstructure debunching.

5. An RF cavity which is used to produced the RF barrier bucket which facilitates the bunch compression required by the Short Pulse Mode.

6. Scintillating screen beam viewers which can be used for beam alignment, emittance measurement through the quadruple scan method, and as an emergency beam stop.

Additionally, a chicane magnet system may also be installed in the electron storage ring to deflect the electron beam towards the experimental regions of future accelerator physics research endeavors, such as to a laser system for an inverse Compton scattering light source or to a wriggler for an FEL.
The ALPHA extraction line contains the octuple magnets responsible for the nonlinear beam spreading and a series of dipole magnets that direct the beam on a winding path towards the radiation test area. Additionally, the extraction line also contains the Faraday cup that is described in the paper. The location of the Faraday cup designed and simulated below is shown in Figure 5.

![Diagram of Faraday cup in extraction line](image)

Figure 5: Proposed Faraday cup position in extraction line.

### 3.4 ALPHA Design Parameters

The Faraday cup described in this paper is intended to be used for accurate beam current measurements only during ALPHA’s Long Pulse (Single Pass) Mode. Thus, the beam parameters used during the design process are determined by the maximum single pulse that can be produced by the injection line linear accelerator. The planned full power beam parameters for ALPHA’s operation are presented in Table 1:
<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron Energy</td>
<td>50.0</td>
<td>MeV</td>
</tr>
<tr>
<td>Peak Current</td>
<td>68.0</td>
<td>mA</td>
</tr>
<tr>
<td>Macropulse Charge</td>
<td>272</td>
<td>nC</td>
</tr>
<tr>
<td>Electrons per Macropulse</td>
<td>$1.7 \times 10^{12}$</td>
<td>$e^-$</td>
</tr>
<tr>
<td>Maximum Pulse Length</td>
<td>4.0</td>
<td>µs</td>
</tr>
<tr>
<td>Maximum Repetition Rate</td>
<td>10.0</td>
<td>Hz</td>
</tr>
<tr>
<td>Maximum Average Power</td>
<td>136</td>
<td>W</td>
</tr>
</tbody>
</table>

Table 1: ALPHA Beam Design Parameters

4 Principles of Faraday Cup Design

The information related to Faraday cups in this section has been culled primarily from references [5-15]. Some general aspects of Faraday cup design and operation are discussed in multiple references and, as such, it is difficult to determine the originator for some topics. For this reason, this section only explicitly cites statements that are unique or seminal with the rest of the information being considered common knowledge among the accelerator physics community. Some of the terminology in this paper is perhaps unique or idiosyncratic and is used to help organize some of the concepts relevant to Faraday cup design and phenomena. The intended meaning of new phrases is explained in detail where they are defined.

4.1 Beam Current Measurement

As discussed in the Introduction, beam current monitoring is an essential component of the diagnostics system of any charged particle beam experiment. A number of techniques have been developed to accomplish this task [16]. Prominent non-destructive examples include the beam current transformer and the wall current monitor. Beam current transformers typically consist of a toroidal pickup coil through which the charged particle beam passes, inducing a signal that is proportional to the beam current which can be measured and filtered by some associated active or passive electronics. A wall current monitor consists of a resistor connected across a physical gap in the beam pipe wall. The image charges from the charged particle beam induce a potential difference across the longitudinal length of the wall gap that is proportional to the beam current.
and may be read out as the voltage across the resistor.

Both of these methods have distinct strengths and weaknesses in terms of mechanical complexity, compatible frequency range, and measurement sensitivity. Additionally, along with all non-destructive current monitoring schemes, they share the advantage of not disrupting beam operation. For linear accelerators, this implies that the current measurement can be made upstream of the target or collision point while, for periodic accelerators such as synchrotrons or storage rings, this translates to the ability to maintain continuous operation while monitoring the beam current. However, all non-destructive schemes share the disadvantage of yielding only a relative measure of the beam current and, consequently, requiring an absolute current measurement for calibration. This absolute calibration measurement is often accomplished with the use of a Faraday cup.

4.2 The Faraday Cup Concept: Basic Physics

A schematic design of a very simple Faraday cup system is shown in Figure 6. The most basic design consists solely of an electrically conductive material in the shape of a cup, typically with cylindrical symmetry, which is placed in the path of the charged particle beam whose current is to be measured. Ideally, all the particles in the beam are absorbed by the Faraday cup, thereby inducing a charge on the cup that can be read out as a current with the appropriate electronics. In this theoretically ideal case, the current passing through the Faraday cup is a direct continuation of the charged particle beam, i.e. the Faraday cup by itself acts as a resistor, so the current read out from the Faraday cup is exactly equal to, and is therefore a direct measurement of, the beam current. (It is important to note that the impedance of a physically realized Faraday cup may also feature a capacitive component, particularly when the Faraday cup is surrounded closely by the wall of a vacuum chamber or by conductive shielding material. As discussed below, this is not an issue for the ALPHA Faraday cup which is kept outside of beam vacuum and is shielded by concrete. The capacitance formed between the Faraday cup and the beam pipe or between the Faraday cup and the structural rebar in the concrete should be negligible.)

Although this analogy between a Faraday cup and a resister is useful at illustrating the essential principle behind Faraday cup operation, it unduly simplifies the interface between the charged
particle beam and the Faraday cup, hiding the underlying complexities of the interactions involved with the transfer of charge from the beam line vacuum to the conducting material of the Faraday cup. A more detailed examination of the physical processes involved in the scattering of the incident charged particle beam with the metal Faraday cup target can provide useful insight during the Faraday cup design process. The discussion here is somewhat restricted to electrons which are the particles of interest for this study, although protons, positrons and heavy ions will exhibit many similar behaviors, albeit with somewhat different physical underpinnings and within different energy regimes.

The interaction of electrons with matter is a rich and complex subject and many books and papers have been written on the issue [17-21 and the references within]. The treatment here follows the development in several of these references, but it is necessarily brief and cursory, presenting primarily those results which directly relate to the physics involved with Faraday cups. Figure 7 shows the relative importance of some of the prominent energy loss mechanisms that are involved during electron and positron scattering in lead [17].

For low energy electrons, we see that ionization is the dominant energy loss pathway with Møller scattering (scattering between electrons) also being present. Ionization can be an important process in Faraday cup physics due to the secondary electron production that can occur through collisions with valence shell electrons within the Faraday cup target metal. However, as discussed below, this is not the primary mechanism involved with the electromagnetic particle cascade that
is observed with high energy charged particle scattering. For very low energy electrons, several other energy loss mechanisms including Auger electron production, characteristic x-ray production, and phonon excitation due to lattice structure interactions can become prevalent. While these are interesting and importance phenomena, they are of little consequence to the design of particle accelerator Faraday cups which typically involve electron beams with energies of at least tens of MeV’s. However, lower energy Faraday cups do find use in certain fields such as materials science and plasma physics, and in industry [22-24].

For high energy electrons, bremsstrahlung radiation caused by Coulomb scattering becomes the extremely dominant energy loss mechanism. Coulomb scattering is an elastic process whose scattering events deflect the angle of the incident electrons’ trajectory through an interaction with the Coulomb potential of the constituent atoms within the Faraday cup’s metal target. Coulomb scattering is the process responsible for the large angular deflections that can lead to backscattered electrons. This process is elastic only in the sense that the interaction between the incoming electron and the target Coulomb potentials is itself elastic. Since the electron is a very low mass charged particle, the acceleration that accompanies the angular deflection results in a significant energy decrease due to bremsstrahlung radiation. As is evident from Figure 7 [17], for high velocity
electrons and a high-Z target material, this energy loss mechanism dominates to the extent that the penetration depth of the incident electron in the target metal can be well approximated by the radiation length which is derived based purely on bremsstrahlung energy loss considerations [17]. This is further discussed in the “Penetration Losses” section below.

The bremsstrahlung radiation caused by Coulomb scattering produces high energy secondary photons. Following the convention of differentiating x-rays and gamma rays based on their origin, this paper refers to these secondary photons as x-rays because they are produced by extranuclear electrons, despite have energies up to tens of MeV’s and beyond, depending on the energy of the incident electron beam. In this energy regime, the interaction between the secondary x-rays and the metal of the Faraday cup can lead to electron-positron pair production [17]. In accordance with charge conservation considerations, this process should not directly affect the long-term or integrated accuracy of the beam current measurement, although instantaneous variations in the comparative absorption efficiencies of the electrons versus the positrons may lead to an increase in the signal shot noise for low beam current amplitudes.

The electrons and positrons which are formed by pair production in bremsstrahlung x-rays possess energies lower than the energy of the particle from which the bremsstrahlung x-ray originated. However, the resulting electron and positron energies can still be above the threshold to produce a new bremsstrahlung x-ray which can undergo electron-positron pair production again. This cyclical formation of secondary radiation results in the electromagnetic cascade of charged particles and photons that is ubiquitous in the interaction of high energy charged particle beams with matter. Figure 8 shows the simulated electron, photon, and energy flux of 30 GeV electrons scattering in iron as a function of longitudinal depth in terms of radiation length (to be defined in the “Penetration Losses” section below) [17]. Note that the electron flux curve lags behind the photon curve, which is an indication that the production of bremsstrahlung x-rays with energies too low for electron-positron pair production occurs near the end of the electromagnetic cascade.

The energy value which represents the shift in energy loss pathway dominance from ionization processes to the production of bremsstrahlung radiation is termed the critical energy $E_c$, which is approximated by
where $Z$ is the atomic number of the target material. (Some references, e.g. [17], instead define the critical energy to be the point at which the ionization per radiation length equals the electron energy. While this difference leads to a small correction in the value of $E_c$, the definition above allows the use of the well known Equation 1 as an approximation of $E_c$.) Since the electromagnetic cascade is propagated by the bremsstrahlung/electron-positron pair production cycle rather than by ionization processes, the critical energy helps to define the properties of the electromagnetic cascade penetration. In particular, the $Z$-dependence in Equation 1 serves as a partial justification for using high-$Z$ materials in Faraday cups and shielding. Table 2 shows the values of $E_c$ for some common Faraday cup elemental materials. The first data column contains the critical energies calculated from Equation 1 while the second data column contains the critical energies tabulated by the Particle Data Group [25].

$$E_c = \frac{800 \text{ MeV}}{Z + 1.2}$$ (1)
<table>
<thead>
<tr>
<th>Material</th>
<th>Critical Energy (MeV) From Equation 1</th>
<th>Critical Energy (MeV) From PDG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lead</td>
<td>9.62</td>
<td>7.43</td>
</tr>
<tr>
<td>Copper</td>
<td>26.59</td>
<td>19.42</td>
</tr>
<tr>
<td>Molybdenum</td>
<td>18.52</td>
<td>13.85</td>
</tr>
</tbody>
</table>

Table 2: Critical Energy Values for Common Elemental Faraday Cup Materials

4.3 Faraday Cup Current Loss Sources

The goal a Faraday cup is to completely absorb all of the charged particles in the incident beam and to fully encapsulate the electromagnetic cascade so that the charged absorbed by the Faraday cup will accurately and directly correspond to the charge in the incident beam. However, in practice, this is not trivial and there are a number of sources of current loss that can lead to erroneous measurements and need to be considered during the design process. These include:

1. Penetration Losses
2. Backscatter Losses
3. Current Leakage Sources

These potential sources of error and some possible solutions are discussed in greater detail below. We will find that avoiding penetration losses sets the size scale of the Faraday cup, avoiding backscatter losses requires some clever design ideas, and avoiding current leakage sources is simply a matter of proper design of the physical apparatus.

For reference in this section and beyond, Figure 9 depicts the general design of the Faraday cup considered in this paper.

4.3.1 Penetration Losses

To a large degree, minimizing the penetration losses in a Faraday cup presents the most important consideration during the design process. Aside from a catastrophic current leakage source which should be easy to avoid, current loss through partial penetration of the incident charged particle beam through the Faraday cup beam stop region or through insufficient encapsulation of
the electromagnetic cascade presents the largest possible error in the measurement of the beam current. Furthermore, avoiding current measurement errors through penetration losses is simply a matter of increasing the size of the beam stop region of the Faraday cup. Since it is conventional to place a Faraday cup at a beam dump position, most accelerator systems will have ample space and structural support to include a Faraday cup with adequate dimensions to avoid severe penetration losses (although schemes that involve placing a Faraday cup within a beam line pipe or similarly confined spaces do exist, especially for the low energy Faraday cups that are not described here). Additionally, the cost of the materials used to construct Faraday cups is normally low enough in comparison to other accelerator components that budgetary concerns should not be a major issue.

On the other hand, an excessively large beam stop region can become wasteful and a hassle to construct and install. Therefore, an optimized Faraday cup design should be large enough to capture the desired percentage of the electromagnetic cascade, and no larger. Determining the optimal dimensions of a Faraday cup as shown in Figure 9 to avoid penetration losses involves selecting the appropriate beam stop region length and the Faraday cup radius. Both of these can be approximated using analytical and empirical formulas, as demonstrated below, or they can be computationally simulated, which is described in the “Faraday Cup Simulations” section.
The size scale of the beam stop region length is determined by the radiation length \( X_o \) of the Faraday cup material. The radiation length is defined as the mean distance through which a high energy electron loses all but \( 1/e \) of its initial energy through bremsstrahlung radiation [17], so that

\[
\frac{E}{E_0} = e^{-d/X_o} \quad \text{or} \quad d = X_o \ln \left( \frac{E_0}{E} \right) \tag{2}
\]

where \( E/E_0 \) is the fractional electron energy remaining and \( d \) is the distance travelled in the target material. Equivalently, the radiation length is equal to \( 7/9 \) of the mean free path for electron-positron pair production by high energy x-rays. The radiation length is given by

\[
\frac{1}{X_o} = 4\alpha r_e^2 \frac{N_A}{A} \left\{ Z^2 [L_{\text{rad}} - f(Z)] + Z L'_{\text{rad}} \right\} \tag{3}
\]

where \( \alpha \) is the fine structure constant, \( r_e \) is the classical electron radius, \( N_A \) is Avogadro’s number, \( A \) is the atomic mass of the target material, and \( Z \) is the atomic number of the target material.

For \( Z > 4 \), \( L_{\text{rad}} \) and \( L'_{\text{rad}} \) are given by

\[
L_{\text{rad}} = \ln (18415 Z^{-1/3}) \quad \text{and} \quad L'_{\text{rad}} = \ln (1194 Z^{-2/3}) \tag{4}
\]

The function \( f(Z) \) is an infinite series, but it may be represented to a good degree of accuracy by

\[
f(Z) = a^2 \left[ (1 + a^2)^{-1} + 0.20206 - 0.0369a^2 + 0.0083a^4 - 0.002a^6 \right] \tag{5}
\]

where \( a = \alpha Z \). A simpler expression for the radiation length which is more explicitly in terms of the atomic number \( Z \) is

\[
X_o = \frac{716.4 A}{Z(Z+1) \ln (287/\sqrt{Z})} \left[ \frac{g}{cm^2} \right] \tag{6}
\]

This formula is accurate to within 2.5% for all elements except helium. Table 3 presents the radiation length of some common Faraday cup elemental materials in terms of physical length, i.e. the quotient of the radiation length in units of g/cm\(^2\), the customary unit of mass thickness, and
<table>
<thead>
<tr>
<th>Material</th>
<th>Radiation Length (cm) From Equation 6</th>
<th>From PDG</th>
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<tbody>
<tr>
<td>Lead</td>
<td>0.5536</td>
<td>0.5612</td>
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<td>Copper</td>
<td>1.469</td>
<td>1.436</td>
</tr>
<tr>
<td>Molybdenum</td>
<td>0.9845</td>
<td>0.9594</td>
</tr>
</tbody>
</table>

Table 3: Radiation Length Values for Common Elemental Faraday Cup Materials

the material density. Similarly to Table 2, the first data column contains the radiation length values as calculated from Equation 6, while the second data column contains the radiation length values as tabulated by the Particle Data Group [25].

For small values of $E/E_o$ in Equation 2, this fraction approximately corresponds to the proportion of incident particles that escape a Faraday cup of infinite radius with a beam stop region thickness $d$. So, for example, a beam stop region thickness of 7 radiation lengths yields a charge absorption of about 99.9%. In practice, increases in this dimension are fairly easy to manage since the mass of the Faraday cup increases only linearly with the beam stop region thickness.

The radiation length describes the fractional energy loss of an electron beam as a function of the longitudinal penetration depth. However, due to the angular deflections inherent to multiple Coulomb scattering, the electromagnetic cascade also exhibits a finite transverse extent. To consider this, the transverse analog to the longitudinal radiation length, called the Molière radius $R_M$, is used. The Molière radius is given by [17]

$$R_M = X_o \frac{E_s}{E_e}$$

(7)

where $E_s$ is the scale energy $\sqrt{4\pi/\alpha m_e c^2} \approx 21.2$ MeV, where $m_e$ is the electron mass and $c$ is the speed of light. The Molière radius helps to describe the energy loss at a radial distance $r$ from the incident beam axis (or, equivalently, the Faraday cup axis, assuming the incident beam hits the face of the Faraday cup head on). The fractional electron energy remaining as a function of the radial distance is approximately

$$\frac{E}{E_o} = 10^{-r/2R_M} \quad \text{or} \quad r = 2R_M \log \left( \frac{E_o}{E} \right)$$

(8)
Of course, this expression is clearly approximate. For example, the electromagnetic cascade radius produced by a real electron beam would clearly have a dependence on the incident beam radius, which is not considered within the Molière radius. For electron beams with radii that are much smaller than the Molière radius, this isn’t a important concern. However, it is common to produce electron beams with radii on the scale of the Molière radius (as discussed in the “ALPHA Project Description” section, the creation of a beam this size for radiation survivability testing is one of the goals of ALPHA), and in this case a correction factor may be necessary. However, Equation 8 is appropriate for an order of magnitude estimate of the electron beam energy loss as a function of radial distance.

Table 4 contains the Molière radii of common Faraday cup elemental materials in terms of physical length. As before, the first data column contains the Molière radius values as calculated from Equation 7, while the second data column contains the radiation length values as tabulated by the Particle Data Group [25].

As with Equation 2 for the radiation length, small values of $E/E_0$ in Equation 8 correspond to the proportion of incident particles that escape a Faraday cup of radius $r$ with an infinite beam stop region thickness. This presents a simple method for estimating the appropriate Faraday cup dimensions for a given material and incident electron beam energy. For a given maximum penetration lose error, values of the beam stop region thickness and the Faraday cup radius should be selected so that the sum of the longitudinal penetration loss error and the transverse penetration loss error is less than the maximum penetration loss error:

$$E_{tot} > E_{long} + E_{trans}$$ (9)
Figure 10 shows the longitudinal and transverse penetration errors for a large range of length parameters. We see that the longitudinal penetration error decreases more rapidly than the transverse penetration error and it quickly approaches the point where an increase in the beam stop region thickness has little effect on the total penetration error. Additionally, since the mass of the Faraday cup scales as the square of the Faraday cup radius, we see that the transverse penetration error is also more difficult to mitigate from the standpoint of minimizing the Faraday cup weight. Of course, as mentioned above, it is typically not difficult to support a Faraday cup of adequate dimensions to allow for a very small total penetration error.

Another common method of determining the appropriate dimensions of a Faraday cup involves the use of an equation derived in perhaps the most cited reference on Faraday cup design [5]. Consider a Faraday cup consisting of graphite and lead in the design shown in Figure 9. If the graphite plug consists of $t$ radiation lengths of graphite and the beam stop region consists of $x$
radiation lengths of lead, then the incident electron loss by penetration is approximated by

\[
f_p = 2E_0 \left(1 - \frac{Dt}{E_0}\right) \exp \left[-\frac{1}{4.25} \left(x - \log \frac{E_0}{185}\right)\right] \%
\]  

(10)

As opposed to the analysis above, this equation considers the use of a graphite plug in its description of the penetration loss error (in this paper, the use of graphite is considered a backscatter loss error reduction technique and is discussed in the “Backscatter Losses” section below). Although this equation does not speak to the appropriate value of the Faraday cup radius, it has been used frequently as a starting point during the Faraday cup design process.

### 4.3.2 Backscatter Losses

Electron backscattering occurs when an electron is scattered or produced such that its trajectory forms an angle with the incident electron beam axis that is greater than 90°. Electrons emitted from the Faraday cup in the backscatter direction come from two primary sources:

1. Coulomb backscattering

2. Electron-positron pair production in the backscatter direction

Coulomb backscattering occurs when an incident electron comes very close to scattering off the center of the Coulomb potential produced by the constituent molecules of the Faraday cup target material. This is directly analogous to the observation of alpha particles backscattering off a gold foil in Ernest Rutherford’s famous experiment. As discussed above, while the Coulomb scattering event leading to a backscattered electron is itself elastic, backscattered electrons experience the highest potential accelerations achievable through Coulomb scattering and therefore lose the largest proportion of their initial energy through bremsstrahlung radiation.

Although the bremsstrahlung radiation produced when the incident electron beam interacts with the Faraday cup target material is forward directed to some extent, it may be considered to be isotropic to a first order approximation when the beam stop region thickness is large enough (several radiation lengths, say). It follows that, in this case, the direction of the electron-positron
pair production instigated by high energy bremsstrahlung x-rays is also isotropic to a first order approximation. For this reason, electron-positron pair production will yield some electrons in the backscatter direction that possess an energy lower than the incident electrons. Of course, electron-positron pair production will also yield some positrons in the backscatter direction that could also escape. On average, the number of electrons and positrons that escape through this mechanism should be equal, so this should not affect the integrated incident electron beam current measurement, though, as mentioned before, this could lead to an increase in the signal shot noise if the number of electrons per incident pulse is small enough.

Backscattering losses are in fact rather rare. If no preventative measures are taken to avoid the escape of backscattered electrons, the current loss will represent roughly 5% of the incident electron beam current. This low lose rate is due both to the inherent rarity of Coulomb backscattering, and also due to the fact that an electron which is backscattered within the Faraday cup beam stop region will often scatter again before escaping the target material. These subsequent scatterings will often lower the energy of the backscattered electron to the point that it is effectively absorbed by the Faraday cup and is unable to escape the target material. Furthermore, the possibility exists that a backscattered electron may be re-backscattered back into the forward direction. Due to these reasons, backscattered electrons that escape the Faraday cup tend to be produced near the surface of the target material, which is a relatively rare event. This also helps to explain why electron backscattering is more common for lower energy electron beams.

Nevertheless, since Faraday cups have the potential to provide extremely accurate direct current measurements, it is in the Faraday cup designer’s best interest to consider methods to mitigate the current measurement error due to backscatter losses. A number of clever backscatter loss reduction techniques have been developed. They generally fall into the following categories:

1. Geometry-based techniques
2. Cushion-based techniques
3. Electromagnetic techniques
4. Window-based techniques
The goal of geometry-based backscatter reduction techniques is to choose a Faraday cup shape in such a way that backscattered electrons that escape the beam stop region are less likely to escape the neck region. This idea suggests the genesis of the Faraday “cup” concept itself. Including a neck on top of a solid beam stop introduces the possibility that electrons backscattered off of the beam stop will scatter off the side of the neck before reaching its opening. While the backscatter direction includes all angles greater than 90°, those angles closest to 90° are the most probable, which suggests that scattering off the side of the neck is likely to occur. This probability can be improved by increasing the length of the Faraday cup neck, thereby decreasing the solid angle available for escape. Other more extreme geometry techniques, such as introducing a bend into the neck region, are also possible. However, the simple inclusion of a neck region in the Faraday cup is typically the only geometry-based backscatter reduction technique that is commonly utilized.

Another very common backscatter reduction technique involves the use of a low-Z material that possesses a smaller backscattering cross section than the high-Z Faraday cup target material to “cushion” the incoming charged particle beam. In addition to having a low atomic number, the cushion material should also be conductive to allow any charged particles that are absorbed by the cushion to flow out to the Faraday cup target material so they can be included in the incident beam current measurement. To satisfy these criteria, graphite is very often selected as the cushion material. Copper, having a relatively low atomic number compared to the other common elemental Faraday cup materials lead and molybdenum, is also occasionally used, especially for higher energy electron beams. Determining the optimal thickness of the cushion relative to the neck region length can be a tough problem. Equation 10 can be used to provide a guideline for the thickness of the cushion. However, while increasing the cushion thickness (with a reasonable range) always decreases the number of backscattered electrons within the neck region, it also increases the solid angle available for the backscattered electrons to escape through the neck opening. This competition between the geometric advantage of a longer neck versus the advantageous cushioning effect of a longer graphite plug is not a simple matter to sort out and it is best investigated using computational simulation techniques, as described in the “Faraday Cup Simulations” section.
Electromagnetic techniques involve the use of electric and magnetic fields to either direct the backscattered electrons away from the neck opening or towards the neck wall or the beam stop region. Electric fields may be utilized by either inducing a positive electrical potential within the Faraday cup or by placing a negatively charged electrode near the neck opening. The former case is probably the easiest electromagnetic technique to implement as the requisite electrical connection is already required to read the incident beam current measurement out of the cup. The latter case takes advantage of the decreased energy of backscattered electrons. The electrode voltage may be selected so that in the incident electron beam is only marginally affected by the electric field created by the electrode, while the lower energy backscattered electrons are forcefully repelled. The utilization of magnetic fields typically involves the use of strong permanent magnets, such as an axially polarized rare earth metal ring magnet, placed near or within the neck region of a Faraday cup. In the case of an axially polarized ring magnet, the magnetic field would induce a force towards the neck wall for all backscattered electrons, except for rare backscattered electron travelling at 180° relative to the incident electron beam axis. As a more extreme case, an actual accelerator dipole magnet could be used to direct the backscattered electrons, and the incident electrons as well, towards the Faraday cup neck wall. This method, however, does not seem to be used.

Window-based electron backscatter reduction techniques use a thin solid window placed near the Faraday cup neck opening to inhibit backscattered electrons from escaping. As with the electromagnetic technique of placing a negatively charged electrode near the neck opening, this method also relies on the fact that back scattered electrons possess lower energies than their incident electron counterparts. The window material and thickness should be chosen so that it is essentially transparent to the incident electron beam while representing an impassable obstacle to the lower energy backscattered electrons. To accomplish this, a metal foil is typically used. In this case, the window foil can be isolated from the Faraday cup material and given a negative charge, thereby combining a window-based technique with the electrode technique. Alternatively, a series of baffles, perhaps consisting of metal disks that are thicker than the foil window used to block the neck opening, can be placed within the Faraday cup neck region. These can serve to both scatter the incoming
electron beam into the walls of the Faraday cup while also blocking the escape of backscattered electrons. However, it should be noted that the use of either windows or baffles does introduce the possibility of the production of secondary radiation particles that can be absorbed by the Faraday cup, leading to an error in the incident beam current measurement.

All of these potential electron backscatter reduction techniques were considered during the initial design process of the ALPHA Faraday cup. Ultimately, it was determined that only the inclusion of a neck segment on the Faraday cup, as well as the insertion of a graphite plug, would be necessary. These two techniques are the simplest to implement and require no maintenance. Furthermore, as the energy of the incident electron beam increases, leading to an increase in the dimensions of the Faraday cup beam stop region to prevent penetration losses, increasing the length of the neck region and the thickness of the graphite plug becomes easier to do. In the end, these techniques are so effective that the other techniques provide little additional benefit.

4.3.3 Current Leakage Sources

Leakage current sources take the form of any electrically conductive route from the Faraday cup to a different electric potential. This is most commonly formed by the support structures to hold up the Faraday cup within its enclosure, if such an enclosure is used. To avoid this, only insulating materials should be used where appropriate, such as plastic screws or ceramic legs. Additionally, since the electron beam hitting the Faraday cup may lead to sputtering, the insulating supports should be periodically cleaned to avoid a conductive path being formed by a thin deposited layer of the conductive Faraday cup material on the insulating materials.

Another potential current loss source similar to a leakage current is the presence of a cycling water cooling system. Some type of cooling system can be necessary in higher energy beam facilities, especially if the Faraday cup is kept under a vacuum, where the lack of heat conduction through air makes ambient cooling more difficult while the temperature of the Faraday cup should be minimized to avoid excessive outgassing. However, the use of a water cooling system does introduce an error in the beam current measurement as some of the scattering electrons will be absorbed and swept away by the water flow. Luckily, as indicated by the ALPHA maximum design beam parameters in Table
1, heating is not an issue in the ALPHA Faraday cup, even under several worst-case assumptions.

4.4 Faraday Cup Electronics

In order to read out the incident charged particle beam current from the Faraday cup, the conductor of a transmission line cable will need to be placed in electrical contact with the Faraday cup target material. It is important that this connection be very solid so that it can be reliably maintained during beam operation. In addition to relaying the beam current measurement, the connection also prevents charge buildup within the Faraday cup. Excessive charge buildup can lead to electrical arcing which can present a safety concern as well as possibly damaging any nearby instruments. If the Faraday cup will be constructed using metal casting techniques, as is planned for the ALPHA Faraday cup, a very strong electrical connection can be made by simply inserting the conductor from a transmission cable into the backside of the Faraday cup beam stop region.

Once an adequate electrical connection has been formed from the Faraday cup, the processing of the beam current signal needs to be considered. The signal can be read out using any number of instruments, such as an ammeter or an oscilloscope. It should be noted that the beam current signal will exhibit artificial spikes at the beginning and end of the signal. Assume the incident electron beam pulse forms a square wave. In the middle of a pulse, the electron beam may be considered to be electrostatic before hitting the cup since there are no accelerating charges. But once the beam hits the cup, a nonzero longitudinal derivative in the radial electric field is introduced leading to a nonzero curl in the electric field. This implies that a magnetic field, and thus a current in the Faraday cup, is induced. For an ideal square wave, the current would be infinitely large in amplitude but infinitely short in time. The ALPHA beam pulses will have rise times and fall times between 1-10 ns, leading to large spikes at the beginning and end of the current signal. Since this spike is a quickly varying signal, it should be eliminated by the inclusion of the low pass filter in series with the transmission line.
5 Faraday Cup Simulations

Many questions concerning particle transport have been for a long time considered either impossible or impractical to answer using analytical techniques. This realization, along with the advent of modern computing capabilities, formed the impetus for the development of stochastic Monte Carlo techniques at Los Alamos in the 1940’s [26]. Since then, the field of computational particle simulations has advanced considerably, both in terms of available hardware resources and also in terms of improvements in software. Geant4 perhaps stands at the forefront of modern particle simulation software. Written using thoroughly modern object-oriented software engineering techniques, Geant4 allows its application developers to create simulations of complicated geometries with relative ease. It was originally created to model the large particle detectors at CERN’s LHC, but it has since spread to many fields, including space science, medical physics, and accelerator physics, due to its ease of use and high level of extensibility.

Near the beginning of the design process for the ALPHA Faraday cup, it quickly became apparent that the formulaic design methods described above were inadequate to optimally design the Faraday cup and to predict its behavior with any degree of accuracy. So it was quickly decided that creating a Geant4 simulation of the Faraday cup would be a natural and potentially very useful exercise. The sections below describe the simulation process and report the relevant results. Unless otherwise noted, the simulations discussed below used an incident 50 MeV electron beam consisting of 100,000 particles with a Gaussian profile with a variance selected to yield a full width at half maximum (FWHM) of 1 cm (however, multiple simulation studies have shown that the results the simulation are only weakly dependent on the incident electron beam radius, up to a point). The physics lists, which specify the physical phenomena and cutoff energies used in the simulation, selected for this study were compiled by SLAC and are accurate down to 100 eV.

5.0.1 Initial Design Studies

As was discussed in previous sections, the first step in the Faraday cup design process involves determining the general dimensions required to avoid excessive penetration losses. This is what
sets the scale of the Faraday cup. The first concern is the calculating the required thickness of the beam stop region. To determine this, a series of simulations were run wherein a cylinder consisting of a common Faraday cup material with a constant radius of 36”, chosen to be much larger than the electromagnetic cascade radius, and a variable thickness was exposed to the incident electron beam. Figure 11 shows the quantitative results of this simulation series. Based on these results, it was decided that lead is the preferred Faraday cup target material and that the use of 8” for the beam stop region thickness would be adequate but not excessive. Figure 12 presents a 200 particle sample of the simulation for 8” of lead.

![Figure 11: Forward Penetration Proportion Thickness Study](image)

The next design step involves determining the Faraday cup radius required to fully encapsulate the electromagnetic cascade. To do this, another series of simulations was conducted using an 8” thick lead cylinder with a variable radius. The quantitative results of this study are shown in Figure 13. From this we see that a value for the Faraday cup radius of around 3” is sufficient. In order to conform to a standard pipe size, a Faraday cup radius of 3.0325”, which corresponds to the inner
(a) Overview. Electrons are red, positrons are blue, and neutrons are green.

(b) Close Up. Electrons are red, positrons are blue, and neutrons are green.

Figure 12: Forward Penetration Proportion Thickness Simulation Sample
diameter of a 6” Schedule 40 pipe, was chosen. A 200 particle sample of the simulation for this Faraday cup radius is shown in Figure 14.

![Figure 13: Side Penetration Proportion Thickness Study](image)

Next, a constraint of 12” was placed on the total length of the Faraday cup core, which implies a neck region length of 4”. This constraint was chosen as a reasonable maximum that yields entirely adequate results. The next series of simulations was designed to determine the optimal graphite plug thickness by incrementing the length of the graphite plug placed within the neck region of the Faraday cup. The quantitative results are shown in Figure 15. Notice that the competition between the beneficial graphite plug cushioning effect and the detrimental increase in solid angle available for backscattered electron escape is apparent. Based on this data, a graphite plug thickness of 3” is clearly optimal. With this, we have determined the basic Faraday cup dimensions. Figure 16 presents a 200 particle sample of this configuration.
(a) Massive particle trajectories. Electrons are red, positrons are blue, and neutrons are green.

(b) Photon trajectories

Figure 14: Side Penetration Proportion Thickness Simulation Sample
5.0.2 A Realistic Design

A schematic of the realistic design for the ALPHA Faraday cup as it will be constructed is shown in Figure 17. Since the ALPHA beam line is designed to achieve very good ultrahigh vacuum pressures down to $10^{-11}$ torr, it is advantageous to place the Faraday cup outside the main vacuum to simplify the design and because lead has a high outgassing rate. For this reason, the beam line pipe is blanked off with a 0.18 mm beryllium vacuum window and is partially inserted into the Faraday cup neck region. A separation distance of $1/2$" between the beam line pipe window and the graphite plug is assumed for simulation purposes, but this value is adjustable.

The design suggests its construction procedure. The front and back $1/2$" end plates should be welded on to the 12" Schedule 40 6" pipe, forming the Faraday cup case. A 2.5" diameter graphite rod can be inserted into the hole in the front end plate, sealing it while molten lead is poured into the Faraday cup case through the hole in the back end plate. Once the case has been filled with
(a) Massive particle trajectories. Electrons are red, positrons are blue, and neutrons are green.

(b) Photon trajectories

Figure 16: Graphite Plug Simulation Sample
lead, the transmission cable conduction can be inserted into the hole in the back end plate, creating the required electrical connection. Once the molten lead has set, a hole can be drilled through the graphite rod in the front end plate, forming the Faraday cup neck region. Casting the lead core around the graphite rod will help to ensure the best possible electrical conduction between the two components.

Figure 18 shows a 200 particle simulation of the realistic ALPHA Faraday cup. A 1,000,000 particle simulation demonstrates that this Faraday cup manages to capture about 99.4% of the incident electron beam.
(a) Massive particle trajectories. Electrons are red, positrons are blue, and neutrons are green.

(b) Photon trajectories

Figure 18: Realistic ALPHA Faraday Cup Simulation
6 Conclusion

The simulations discussed in Section 5, as well as those that were not described, have allowed for a certain high degree of confidence in the proposed ALPHA Faraday cup design that would not have been possible without the use of Geant4. The ability to optimize the design and test its performance before the construction of the device will undoubtedly result in a better design than could be achieved through the analytical and empirical methods described in Section 4, which leave many design questions unanswered. It is a testament to the power of Geant4 to note that simulations described in this paper only scratched the surface of the full breadth of its usefulness.

7 References


